Quantity observations as a path to Pareto improvements*

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This version: September 2012

Abstract

Quantity stabilizations give agents the right or obligation to repeat their earlier trades but otherwise allow for unrestricted trade; they lead to Pareto improvements and Pareto efficiency and can be implemented with information only about traded quantities, not utility or demand functions. Quantity stabilizations have Coasean advantages: by assigning clear property rights they achieve efficiency, while the realization of a Pareto improvement can overcome any opposition to reform. Applications exhibit a trade-off of advantages and drawbacks. In partial-equilibrium settings, quantity stabilizations use relatively little quantity information but obligate agents to repeat their earlier purchases as well as their sales. General-equilibrium quantity stabilizations can let repetitions of purchases be optional, but more quantity information is necessary.

JEL codes: D6, H2, D51, P21

Keywords: Pareto improvements, welfare economics, Slutsky compensations, rent control, planning.

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*Parts of this paper earlier appeared as Cowles Foundation Discussion Paper 1320 and may be downloaded from http://cowles.econ.yale.edu/P/cd/d13a/d1320.pdf.

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1 Introduction

Most of the policy proposals recommended by economists harm at least some agents. In principle, one may award any harmed agent a compensation payment, but limits on information mean that policymakers usually cannot identify losers or calculate how much they need to receive. This paper examines when information about the quantities agents trade ex ante can be used to avoid these problems and achieve a Pareto improvement.

Suppose an economy suffers from some preexisting inefficiency – say the state directs production in some industries. If policymakers can observe agents' initial net trades, they can give agents the right to repeat those trades and the option to make further trades, thus ensuring that no agent ends up worse off. One way to view these policies, which we call *quantity stabilizations*, is as a set of lump-sum transfers that ensure that agents can afford their ex ante trades. Since the transfers are lump sum, the allocation that results will be Pareto optimal as well if markets are complete and competitive.

A simple example, laid out in section 2, applies to the inefficiency caused by rent control. Agents under a quantity stabilization can continue their rent-control leases but they gain the option to sublet their rental properties at market prices. Since renters have an unrestricted right to reallocate properties at market-clearing prices, efficiency is achieved. At the same time the ability of renters to repeat their ex ante occupation of their rental properties guarantees that they will not end up worse off. As we will see, ensuring that property owners do not suffer diminished welfare under a quantity stabilization is more of a problem.

The classical way to reach a Pareto-improving optimum proceeds via the second welfare theorem: the policymaker selects an optimum and uses the prices that support it to set transfers so that agents can exactly afford their designated bundle. But the knowledge needed to identify an optimum and its supporting prices is inaccessible in the extreme. Quantity stabilizations, in contrast, aim only to achieve *some* Pareto-improving optimum. This change of perspective makes the transfers easy to calculate.

We consider three roadblocks that can stand in way of quantity stabilizations. First, in partial equilibrium settings a Pareto-improving quantity stabilization will *require* agents to repeat the purchases they made ex ante (prior to the policy change), and this may be difficult to enforce or be politically infeasible. General-equilibrium quantity stabilizations on the other hand can always be designed so that the repetition of purchases is optional. The second difficulty gives partial equilibrium applications the advantage. Quantity stabilizations require policymakers to know the trades that agents made ex ante of all the goods whose prices are affected by the policy; if this information is missing then consumers must be allowed to buy or sell arbitrary quantities at the prices that ruled prior to any policy change to ensure that a Pareto improvement is reached, and this leeway can bankrupt the firms that trade with consumers, thus making quantity stabilizations infeasible. General-equilibrium quantity stabilizations, where the prices of many goods change, therefore require a great deal of information – less information than the second welfare theorem demands, to be sure, but still too much. In partial-equilibrium applications, in contrast, the necessary quantity information can be modest.

These first two difficulties form the fundamental dilemma of quantity stabilization. In partial equilibrium, informational requirements are reasonable but if buyers cannot be compelled to repeat their purchases the quantity stabilization may not be Pareto-improving, while in general equilibrium, it does no harm to give buyers the right to refuse but informational requirements can be onerous.

The final difficulty is particular to quantity stabilization as a reform tool for planned economies, previously studied by Lau, Qian, and Roland (1997, 2000) in their theory of dual-track reform. In the Lau et al. model, agents are rationed under planning and then under reform have the right to repeat, at the plan's prices, the purchases and sales they made previously. The model is not internally consistent, however, since it does not reconcile the dual system of budget and rationing constraints that agents face under planning. Specifically, agents are not allowed to accumulate money under planning even though generically they will have to do so. The model can be made consistent if the government runs a deficit under planning, but new taxes must close the deficit under any reform and this can jeopardize the achievement of a Pareto improvement.

Even though quantity stabilizations are Pareto-improving, the allocations achieved may not be the most desirable from a welfare point of view: a standard competitive equilibrium with no accompanying compensations might be judged to be normatively superior. On the other hand with a Pareto improvement the vested interests that may otherwise block all change can in effect be bought off. If a laissez-faire competitive equilibrium is the only viable alternative, a grossly inefficient status quo might prevail.

2 Quantity stabilization: basic theory

To present the basics of a quantity stabilization, we begin with an exchange economy and examine when quantity stabilizations are Pareto improving. The set-up covers both partial and general equilibrium settings. The economy's status quo occurs at an initial 'prereform' time period. A 'reform' – that is, a policy change – is then applied to the same economy and we examine when the resulting allocation is a Pareto improvement relative to the prereform allocation.

Each of a finite number of agents i has a preference relation $\succsim_i$ defined over nonnegative consumption bundles of n goods, $x_i \in \mathbb{R}^n_+$, with the associated strict preference relation $\succ_i$. Agent i has an endowment of the n goods $e_i \geq 0$.[1] In any period, i has a budget set B_i of affordable consumption bundles and selects a x_i in B_i such that $x'_i \in B_i$ implies not $x'_i \succ_i x_i$.

In the prereform period, prices for the n goods are $\overline{p}$ and agent i's budget set is $B_i = \{x_i \in \mathbb{R}^n_+ : \overline{p} \cdot x_i \leq \overline{p} \cdot e_i\}$. Let $\overline{x}_i \in B_i$ denote the initial consumption that i selects, and let $\overline{z}_i = \overline{x}_i - e_i$ be the corresponding excess demand. The policymaker observes $\overline{p}$ and $\overline{z}_i$ for each agent i.

We do not need to specify precisely how the initial equilibrium is determined or impose any assumptions on $\succsim_i$. Allocations need not be governed by competitive markets or may suffer from an inefficiency that calls for reform, e.g., the government imposes price ceilings or distorting taxes on some goods. For quantity stabilizations, the most relevant types of inefficiencies are those imposed by government or regulatory policy and that could be removed simply with a change in policy – though a simple removal of a government-imposed distortion will generally harm some agents, which motivates our search for Pareto improvements. Formally our only restriction on the prereform period is the requirement that the

[1] For vectors x and y, $x \geq y$ means $x(k) \geq y(k)$ for each coordinate k, $x > y$ means $x \geq y$ and not $y \geq x$, and $x \gg y$ means $x(k) > y(k)$ for each k.

sum of the initial net trades is feasible, $\sum_i \overline{z}_i \leq 0$.

The economy proceeds to a second period of activity, with preferences and endowments the same as in the initial period. A *quantity stabilization* leads to a new price vector $p \geq 0$ but gives agents the right to repeat their earlier trades at the prices $\overline{p}$ and the obligation to repeat those trades when the other party to an earlier trade insists. Agents may resell any goods they acquire through these repeated trades, but resales occur at the new prices p. When agent i is a net purchaser of good k and the price of k, $p(k)$, rises relative to its prereform price $\overline{p}(k)$, that is, if

$$\overline{z}_i(k) > 0 \text{ and } p(k) > \overline{p}(k),$$

i will want to repeat the trade, and thus earn $(p(k) - \overline{p}(k))\overline{z}_i(k)$ in arbitrage profits. These are i's profits even if i does not consume any of good k since after buying $\overline{z}_i(k)$ units of k at $\overline{p}(k)$, i may sell them at price $p(k)$. When i is a net seller of k and the price of k rises relative to its prereform price,

$$\overline{z}_i(k) < 0 \text{ and } p(k) > \overline{p}(k),$$

the agents who bought from i will invoke their right to repeat their trades. So i experiences a loss of $(p(k) - \overline{p}(k))\overline{z}_i(k)$. The cases when $p(k) < \overline{p}(k)$ are similar. In sum, agent i receives arbitrage profits equal to

$$\sum_k (p(k) - \overline{p}(k))\overline{z}_i(k) = (p - \overline{p}) \cdot \overline{z}_i.$$

Hence i's budget set under a quantity stabilization is given by

$$B_i^{\mathrm{QS}} = \{x_i \in \mathbb{R}_+^n : p \cdot x_i \leq p \cdot e_i + (p - \overline{p}) \cdot \overline{z}_i\}.$$

Since $p \cdot e_i + (p - \overline{p}) \cdot \overline{z}_i = p \cdot \overline{x}_i - \overline{p} \cdot \overline{z}_i$ and $\overline{p} \cdot \overline{z}_i \leq 0$, agent i can replicate his or her prereform consumption, that is, $\overline{x}_i \in B_i^{\mathrm{QS}}$. Hence,

Proposition 1 *No agent is worse off with the allocation achieved in a quantity stabilization* *compared with the prereform allocation.*

Since Proposition 1 involves only an appeal to the affordability of agents' prereform consumption bundles, it requires no assumptions on agents' preferences.

The budget set B_i^{QS} differs from B_i only due to the presence of the lump-sum transfer $(p - \overline{p}) \cdot \overline{z}_i$. Hence the consumer side of the economy under a quantity stabilization is consistent with full Pareto efficiency: any change in any agent's consumption is valued using the same price vector p. Full efficiency will therefore obtain in the absence of distortions (such as externalities, missing markets, and so on).

Quantity stabilizations can be instituted in two ways:

(1) a decentralized set of private trades that agents have the right or obligation to repeat, or

(2) a set of lump-sum transfers issued by the government.

The advantage of method (1) is that quantity stabilization can remain feasible even when policymakers have no immediate knowledge of the $\overline{z}_i$. Of course, if the government has no access to the $\overline{z}_i$ whatsoever, then agents will repudiate their obligations to trade. But if agents can establish the validity of claims about prereform trades in court, say by suing their trading partners if they refuse to honor their obligations to repeat trades, then a decentralized quantity stabilization may still be possible. The mere threat of court action by itself might be sufficient to enforce the repetition of trade.

Note also that method (1) does not have to proceed via a physical transfers of goods; agents i for whom $(p(k) - \overline{p}(k))\overline{z}_i(k)$ is negative could instead hand over $|(p(k) - \overline{p}(k))\overline{z}_i(k)|$ in money to their previous good-k traders to discharge their obligations.

Under method (2), the government in effect transfers to agents the appropriate Slutsky compensations, $(p - \overline{p}) \cdot \overline{z}_i$ to each agent i. If *complementary slackness* obtains ex ante, that is, if $\sum_i \overline{z}_i(k) < 0$ implies $\overline{p}(k) = 0$, then the government can afford the sum of the transfers, $\sum_i (p - \overline{p}) \cdot \overline{z}_i$. To see this, complementary slackness implies $\overline{p} \cdot \sum_i \overline{z}_i = 0$, and hence $\sum_i (p - \overline{p}) \cdot \overline{z}_i = \sum_i p \cdot \overline{z}_i = p \cdot \sum_i \overline{z}_i$. Since we have assumed that $\sum_i \overline{z}_i \leq 0$, we have $p \cdot \sum_i \overline{z}_i \leq 0$. So the sum of the quantity-stabilization transfers, $\sum_i (p - \overline{p}) \cdot \overline{z}_i$, is nonpositive.

Notice how easy the Slutsky compensations are to calculate compared to the Hicksian compensations that transfer to each i exactly the amount of income that would keep i at his/her ex ante utility level. Slutsky compensations generally will not leave agents at the same utility levels, but the errors always overshoot: no agent's utility falls relative to its prereform level.

Suppose that agents prior to reform spend all their wealth, $\overline{p} \cdot \overline{z}_i = 0$ for each i, as agents normally would in a market equilibrium model. Then the lump-sum transfers are very simple: $p \cdot \overline{z}_i$ to each agent i. The government thus does not even need to know the prereform prices $\overline{p}$ to calculate quantity-stabilization transfers. Also, since complementary slackness must obtain when $\overline{p} \cdot \overline{z}_i = 0$ for all i, the government can necessarily afford the transfers in this case. We will see in our application to planning, however, that unlike standard competitive markets it is not unusual for $\overline{p} \cdot \overline{z}_i < 0$ to hold for some i.

Our treatment thus far has been general equilibrium. A partial equilibrium quantity stabilizations affects only a subset K of the entire set of goods $\{1, ..., n\}$. Suppose that prior to reform the government knows the prices and trades only for the goods in K, that is, the government knows $\overline{p}(k)$ and $\overline{z}_i(k)$ for each agent i and each $k \in K$. In the spirit of partial equilibrium analysis, we assume that any reform will affect only the prices of the goods in K: for $j \notin K$, $p(j) = \overline{p}(j)$. The transfer to agent i then reduces to

$$\sum_{k \in K} (p(k) - \overline{p}(k))\overline{z}_i(k).$$

With these transfers, which as before could either be lump-sum payments from the government or the returns from decentralized trades, each agent can again afford his/her prereform trades and hence cannot be worse off. Notice that since the government does not know the trades of non-K goods, it will not know $p \cdot \overline{z}_i$. Hence if the government were to use the lump-sum transfer method ((2) above) for delivering quantity stabilizations, it would not be able to rely on the simpler transfers, considered earlier, that ignore prereform prices.[2]

We have assumed that when a quantity stabilization occurs through decentralized private

[2] A transfer of $\sum_{k \in K} p(k)\overline{z}_i(k)$ need not allow agents to afford their prereform trades. For example, if $K = \{1\}$ and an agent i with $\overline{z}_i(1) < 0$ receives a (negative) transfer $p(1)\overline{z}_i(1)$, then i will not be able to repeat his/her prereform trades.

trading all agents can insist that all of their prereform transactions are repeated. We then say *strong obligations* hold. Strong obligations may be difficult to enforce however. When a good k's prereform price $\overline{p}(k)$ exceeds $p(k)$, prereform buyers suffer losses when forced to repeat their purchases and so have an incentive to deny that they made them prereform. Buyers are also more likely than sellers to have a monopoly of evidence of a prereform transaction – consider anonymous shop receipts – and hence will be in a position to suppress that evidence. It may in addition be a serious political challenge to force buyers to repeat trades.

Example 1 (rent control) To illustrate these difficulties, consider how a quantity stabilization would work as a reform of rent control. Let the goods in the subset $K \subset \{1, ..., n\}$ be apartments subject to rent control. The existing tenant of one of these apartments $k \in K$ has the right to live in k for a rental price $\overline{p}(k)$ that typically would be much lower than the price that other individuals would be willing to pay to live in k. Although this discrepancy implies inefficiency, a simple repeal of rent control is likely to leave many rent-control tenants worse off. From the efficiency point of view, the problem with rent control is Coasean: neither current tenants nor owners can reassign properties freely. Quantity stabilizations maneuver around this roadblock. The simplest way to institute a quantity stabilization is to use the decentralized private trading method ((1) above): the existing tenant of apartment $k \in K$ retains the right to rent k but is allowed to sublet k to another agent. If following reform apartment k can be sublet for a rent of $p(k) > \overline{p}(k)$ then the existing tenant can earn the profit $p(k) - \overline{p}(k)$ or can continue to rent and live in k. If we make the partial-equilibrium assumption that the quantity stabilization does not change the prices of goods not in K, then any tenant can continue to consume his or her prereform consumption bundle and therefore cannot be worse off under the reform. So far, so good. The difficulty is that some apartments in K could end up renting for a price that is lower than their prereform rent-control price, $p(k) < \overline{p}(k)$. This possibility is by no means pathological. Under rent control, the limited availability of the most desirable apartments may force richer tenants to live in less desirable outlying areas (e.g., they must live in Brooklyn rather than Manhattan). The greater market availability of desirable apartments under a quantity stabilization could well drive down the value of less desirable apartments. Moreover if, for some apartment

k, $p(k) < \overline{p}(k)$ then the owner of k will be worse off unless the tenant is subject to strong obligations, that is, if the tenant can be forced to continue to pay $\overline{p}(k)$. Needless to say, such a requirement may be unenforceable if tenants can exit the jurisdiction or may be politically unthinkable.

In principle, the government could compensate any landlords suffering a loss by taxing subletting profits, i.e., taxing the tenants of any k with $p(k) > \overline{p}(k)$. Although the receipts from a such tax might not cover subsidy payments, it is plausible that they would. If subletting profits are taxed proportionally the tax would effectively be lump-sum – the tenant of k will continue his/her rent-control lease if and only if $p(k) > \overline{p}(k)$ – and hence would not lead to an efficiency loss. Also, such a tax could be implemented using only information from preexisting leases; the overall reform plan would therefore remain informationally accessible. ■

As the above example illustrates, the drawbacks of strong obligations may force the government to exempt buyers from the obligation to repeat prereform trades. When only buyers but not sellers have the right to repeat a prereform transaction, we say *weak obligations* hold. Given that buyers have the right to repeat transactions under weak obligations, sellers must repeat prereform transactions whenever buyers insist. But since agents may not be able to repeat all of their prereform transactions, quantity stabilizations with weak obligations are not always Pareto improving. Since buyers will invoke their right to buy good k only if $p(k) > \overline{p}(k)$, agent i's implicit transfer under a quantity stabilization with weak obligations equals $\sum_k \overline{z}_k(k) \max[p(k) - \overline{p}(k), 0]$, which could well be negative.

As we will see, there is no need to impose strong obligations in a general equilibrium setting: a policymaker can always allow obligations to be weak and still achieve a Pareto improvement by simply letting the postreform price of every good be greater than its prereform price, thus ensuring that all purchases and hence all transactions are replicated. The next section will treat this point in more detail. In partial equilibrium, in contrast, most goods are not part of the reform and so this trick is unavailable; as a consequence, it may be that any partial-equilibrium quantity stabilization with weak obligations will fail to be Pareto improving.

Quantity stabilizations in the partial equilibrium model do have one major advantage however: they are easier to implement. In general equilibrium, policymakers need to know all of the prereform trades, an assumption that would seem to apply only to a planned economy where the state dictates all trades, and even then, the assumption is demanding. Furthermore as we will see in the next section governments under planning will almost always have to run a deficit, which during a reform can make quantity stabilizations infeasible or prevent the achievement of a Pareto improvement.

Quantity stabilizations therefore present a trade-off to policymakers. Government knowledge of ex ante trades is vastly more plausible in partial equilibrium settings than in general equilibrium, but in partial equilibrium policymakers may have to force buyers to repeat purchases to ensure a Pareto improvement.

3 The feasibility of quantity stabilizations

While we have shown that the government can afford quantity-stabilization transfers, we have not yet considered when quantity stabilizations are compatible with market clearing. Both feasibility in this sense and whether feasible quantity stabilizations can rely on weak rather than strong obligations are intrinsically general equilibrium topics. Since the prime difficulty facing quantity stabilizations is that the government, due to its lack of information, may have to require firms to conduct trades that will force the firms into bankruptcy, we need to include production in our model.

To allow comparison to Lau et al. (1997, 2000), which first proposed quantity stabilization as a general equilibrium policy, we will show how to read the model so that the ex ante inefficiency is state planning. This interpretation is only one of many possibilities; the model is general enough that the ex ante economy could have private ownership and be suffering from a nonplanning inefficiency instead and quantity stabilizations as a cure for planning face some extra problems that do not arise in other settings. Readers uninterested in planning may therefore want to stick to a private-ownership interpretation.

Many of the inefficiencies that arise in general equilibrium models have an obvious remedy. If some goods are subsidized, the subsidies can be removed; if some sectors are protected from international trade, then trade barriers can be dismantled; and if the economy is planned, the government could distribute its productive assets to consumers, let firms maximize profits and let prices be set by supply and demand. In the absence of other distortions, the first welfare theorem implies that the allocation that results from such policies will be Pareto optimal, but the allocation will generally not be Pareto-improving relative to the status quo. For example, industries that have been sheltered from competition may well use backward technologies and agents may have accumulated specialized skills and resources that can be used only by these technologies. When competition is introduced, the demand for these factors will typically diminish, leaving their owners worse off. Following the second-welfare theorem, the losers in this and other examples could be fully compensated via lump-sum transfers from winners, but policymakers usually will not know enough to calculate these transfers.

Quantity stabilizations can serve as a Pareto-improving reform of general equilibrium inefficiencies but they face a couple difficulties. First, the feasibility of a quantity stabilization hinges on policymakers having an exact record of all agents' prereform trades, a demanding level of knowledge even in planned economies. Without such knowledge the only way to ensure a Pareto improvement is to give consumers wide latitude to buy and sell at prereform prices, and this latitude may well bankrupt the firms that consumers trade with. The second problem is specific to planning. If under planning agents are rationed then they will normally accumulate money; to make up for the resulting shortfall in demand, the government must run a budget deficit. In a reform, the deficit must be closed and the need for revenue can undercut the achievement of a Pareto improvement. On the plus side, general equilibrium quantity stabilizations can make do with weak rather than strong obligations.

The feasibility of quantity stabilizations in general equilibrium also raises some technical points, overlooked in Lau et al. (1997). Quantity-stabilization transfers in effect give agents endowments equal to their ex ante consumption bundle, and those bundles may well be on the boundary of agents' consumption sets (even when their original endowments are interior). The resulting discontinuity in demand can mean that equilibria fail to exist. In regard to Grandmont and McFadden's (1972) design of Pareto-improving transfers for autarkic economies entering world trade, this problem was pointed out by Cordella, Minelli, and Polemarchakis (1999). So, to ensure existence of equilibrium we need assumptions that are somewhat stronger than the norm in general equilibrium theory.

We again assume the economy proceeds through two periods, a first period prior to reform and a second – with the same preferences, endowments, and technology – that will be quantity-stabilized. Pareto improvements are policy changes that do not decrease any agent's welfare in the second period relative to the first. We discuss the two-period interpretation in the conclusion.

3.1 The prereform economy

As in section 2, there are n goods and each consumer i has a utility u_i and endowment e_i.

The government makes an observation of agents' net trades in the first period, prior to any policy reform, but the observation need not be exact; the government might know only that an agent's net trades are less than some upper bound. Letting $z_i(k) = x_i(k) - e_i(k)$ be consumer i's net trade of good k and $\alpha_i(k)$ be either the government's exact observation of $z_i(k)$ or the upper bound on the government observation, we assume that one of the following two conditions must hold

$$z_i(k) = \alpha_i(k) \text{ or } z_i(k) \leq \alpha_i(k) \tag{3.1}$$

for each consumer i and good k. The mixture of equality and inequality constraints can vary both with i and k. When the equality in (3.1) applies, then i is *exactly observed with respect to good* k, while if the inequality holds then i is *partially observed with respect to good* k. And if a consumer i is exacted observed with respect to every good, we simply say that i is *exactly observed*.

Let $\alpha_i = (..., \alpha_i(k), ...)$. We assume that $e_i + \alpha_i > 0$ for each i and that $\sum_i (e_i + \alpha_i) \gg 0$: the government's observations are consistent with agent i consuming a positive amount of some good and with the agents collectively consuming positive amounts of all goods.

In applications to planning, the interpretation of (3.1) is different; the condition instead represents the rationing rules imposed on agent i by the government prior to reform. The inequality constraints would apply to consumption goods that can only be purchased in limited quantities while the equality constraints would apply to factors that agents have to deliver an exact quantity of. Since $\alpha_i(k)$ can be set arbitrarily high, the inequality constraints can also cover goods that in effect have no upper limit on purchases. When discussing planning applications, we will say that i is exactly or partially *rationed* rather than observed and use similar terminology for the economy as a whole. For some good k, it could be that none of the partial rationing constraints bind; then no agent is rationed with respect to k, as when the state cannot monitor individual purchases or when consumer budget constraints by themselves sufficiently limit demand. The constraint levels $\alpha_i(k)$ nevertheless remain important, since under a quantity stabilization they will serve as consumer rights to buy goods at plan prices. Although it is common to suppose that rationing is exact in planned economies, such extreme forms of planning have rarely if ever been attempted in large-scale societies. Consumers in planned economies retain at least some discretion about how to spend their incomes; even if factor deliveries are mandated by the state, some consumption purchases remain both optional and anonymous. Moreover, prices in planned economies usually play at least some role in curbing demand. Under exact rationing, in contrast, prices serve no such function since all net purchases are dictated by the state. The plan could therefore be implemented without prices or any exchange of money.

Returning to the general model, the initial period prereform prices are denoted $\overline{p} \geq 0$. Agent i's excess demand $z_i = x_i - e_i$ must satisfy the budget constraint $\overline{p} \cdot z_i \leq 0$. When z_i satisfies this budget constraint and $x_i = z_i + e_i \geq 0$, we say z_i is *affordable*. In a private-ownership economy, utility maximization will lead budget constraints to hold with equality (consumers will not throw away any of their income). So if observations are exact then $\overline{p} \cdot \alpha_i = 0$ will hold.

The situation is different with exact rationing under planning. Then α_i represents the net trade for i that the state mandates, not an observation of optimizing behavior. In order for prereform equilibria to be internally consistent, i must be able to afford the state's instructions. We therefore require that

$$\overline{p} \cdot \alpha_i \leq 0. \tag{3.2}$$

But it would be a knife-edge case for $\overline{p} \cdot \alpha_i$ to equal exactly 0: the exact rationing levels α_i dictated by the state and $\overline{p} \cdot \alpha_i = 0$ place independent restrictions on i that will not normally be mutually consistent. So in generic cases of exact rationing we will have $\overline{p} \cdot \alpha_i < 0$, meaning that consumers in the prereform economy accumulate money. As we will see, this poses an obstacle for quantity stabilizations. Notice that when z_i satisfies (3.1) then (3.2) implies that z_i is affordable.

Prereform production is organized by a finite set of firms, which under planning could be owned by the state. Each firm j has a constant-returns-to-scale production set Y_j, which gives the net productions that are feasible for j. Firm j's prereform net production is γ_j. In a private-ownership economy, γ_j maximizes j's profits. Due to constant returns or because the state owns the firms under planning, there are no operating profits to distribute to consumers.

The government's observations of consumer behavior explicitly or implicitly imply observations of firm behavior. Consumers in the aggregate must make net purchases, or in the case of partial observations potentially make purchases, that equal the exact or potential net sales of the firms. The sum of the exact or potential net purchases for goods produced by firm j will be labeled β_j. We will say that *all agents are exactly observed* if $\gamma_j = \beta_j$ for each firm j and (3.1) holds with equality for all consumers i and goods k.[3] Since the government's observations of consumers and firms must be consistent, we require that

$$\sum_i \alpha_i(k) = \sum_j \beta_j(k) \tag{3.3}$$

for each good k. There can also be observations of inter-firm (or even inter-consumer) transactions but these cancel out in the aggregate.

Prior to reform, a firm j might lose money $\overline{p} \cdot \gamma_j < 0$. Since firms have the option of shutting down (choosing 0 as a production), the primary cases where firms lose money occur when the government mandates firm behavior. If a firm j makes a loss then the government must subsidize it to keep it afloat, that is, give j a transfer that we label τ_j.

Definition 1 *A prereform equilibrium is a price vector $\overline{p}$, a (z_i, α_i) for each consumer i, and a (γ_j, β_j) for each firm j such that (3.1) - (3.3) are satisfied for each consumer i and good k (that is, observation constraints are satisfied and government observations are consistent)*

[3] We will have no need to consider partial observations of firms.

and the following conditions – material balance, feasibility of the γ_j, and nonnegative profits – are satisfied:

$$\sum_i z_i = \sum_j \gamma_j \tag{3.4}$$

$$\gamma_j \in Y_j \text{ and } \overline{p} \cdot \gamma_j + \tau_j \geq 0 \text{ for each firm } j.^4 \tag{3.5}$$

For the sake of generality, Definition 1 makes no mention of optimization but it is consistent with each consumer being a utility maximizer and each firm being a profit maximizer. The generality ensures for example that, in applications to planning, profit maximization need not determine the choice of $\gamma_j \in Y_j$.

When consumers accumulate money balances, which as we have pointed out is generic under planning with exact rationing, the state will run a deficit: it will issue positive net credits to firms in the amount $\sum_j \tau_j > 0$. When consumers accumulate money, their withdrawal of purchasing power would in the absence of subsidies lead firm profits to be negative in the aggregate. So, for the economy's firms to break even, the government must in sum pay out positive subsidies.[5] Call the government budget *balanced* if $\sum_j \tau_j = 0$. In a private-ownership economy, where consumers are not rationed, a prereform equilibrium is perfectly compatible with a balanced government budget. But in a planned economy with exact rationing, a balanced budget would be a fluke.

In the Lau et al. (1997) model of planning, exact rationing obtains but there is no government sector to pay out subsidies. Consequently, in the generic case where consumers accumulate money, prereform ('planning') equilibria will not exist in their model.[6]

[4] As a matter of interpretation, it is natural to suppose in a private-ownership economy that for any firm j that makes a loss j's transfer exactly equals its loss, $\tau_j = -\overline{p} \cdot \gamma_j$; otherwise j would have profits to distribute to consumers. Also, in a private-ownership economy the government must tax consumers an amount equal to its total subsidy payments $\sum_j \tau_j$. Since it is only agents' prereform net trades that are pertinent to quantity stabilizations, we leave the details of how revenue is raised unspecified.

[5] Formally, suppose that at least one consumer h has excess money balances: $\overline{p} \cdot z_h < 0$. Since $\overline{p} \cdot z_i \leq 0$ for each consumer i, summing across consumers yields $\overline{p} \cdot \sum_i z_i < 0$. Since (3.4) implies $\overline{p} \cdot \sum_i z_i = \overline{p} \cdot \sum_j \gamma_j$, it must be that $\overline{p} \cdot \sum_j \gamma_j < 0$. Hence, by (3.5), $\sum_j \tau_j > 0$.

[6] If firm profits and losses go to consumers, which is how Lau et al. (1997) model firms under planning and if Π denotes the sum of firm profits, then we have $\overline{p} \cdot \sum_i z_i < \Pi$ when consumers accumulate money. But (3.4) implies $\overline{p} \cdot \sum_i z_i = \overline{p} \cdot \sum_j \gamma_j = \Pi$. Lau et al. avoid this problem by assuming that planned quantities happen not to let consumers accumulate money. This problem is separate from the technical point (see footnote 9) that we need utilities to be increasing in order to show that equilibria exist.

The feasibility of various reforms depends on the government's information about the exchanges that take place prior to reform. Our assumption is that the government acquires information about exchanges solely from the α_i and β_j parameters; these numbers do not have to be immediately known, but we suppose the government can verify claims about them. The government therefore cannot determine an agent i's precise purchases of good k when i is only partially observed when respect to good k. It would be more realistic to give the government other sources of information, but all that matters is that there are some trades about which the government is not fully informed.

Although we have let planning serve as the leading example of inefficiency, the model can cover many other varieties. We have not specified how production decisions are made; hence they could be made by profit-maximizing producers facing a distortion, e.g., from externalities or commodity taxation.

3.2 Quantity stabilization

When a quantity stabilization is used to eliminate an inefficiency, any productive resources controlled by the government, such as state-owned firms, are distributed to individuals and any government restrictions on firm profit-maximization are removed. A quantity stabilization thus shares common ground with classical economic advice. But in addition agents retain certain rights and obligations to repeat their prereform trades at the prices $\overline{p}$. In contrast to the partial-equilibrium case discussed in section 2, achieving a Pareto improvement will not require strong obligations where agents are forced to repeat their prereform purchases.

When all agents are exactly observed, the government knows all of the agents' prereform trades and can give agents the right to repeat those trades at the prices $\overline{p}$. But when observations are partial, the state's only viable alterative is to give agents the right to buy or sell the quantities given by the prereform upper bounds on actual trades, α_i for consumer i and β_j for firm j.

Given its observation levels, the government has various options regarding which trades it obligates agents to make. When obligations for a good k are strong, all consumers and firms have the right to buy or sell good k at price $\overline{p}(k)$ up to their observation levels $(\alpha_i(k)$

or $\beta_j(k)$). So, if $\alpha_i(k) > 0$ then consumer i can purchase up to $\alpha_i(k)$ units of good k at price $\overline{p}(k)$ and if $\alpha_i(k) < 0$ then i can sell up to $|\alpha_i(k)|$ units of k at price $\overline{p}(k)$. Obligations for good k are weak if all agents have the right to buy k up to their observation level at price $\overline{p}(k)$ but do not have any corresponding right to sell k. Of course if obligations for k are weak, then the potential sellers of k – the agents with $\alpha_i(k) < 0$ or $\beta_j(k) < 0$ – are obligated to sell k when buyers of k invoke their right to buy. We say that *obligations are strong overall* (resp. *weak overall*) if obligations for all goods are strong (resp. weak). We assume that every good is subject to either weak or strong obligations. Although it might seem that weak obligations could allow the economy to end up at an equilibrium that is not Pareto improving, it turns out that exact observations and budget balance by themselves guarantee that Pareto-improving quantity stabilizations are feasible (see Proposition 3).

Consumers who are partially observed with respect to some good k in the prereform economy may well be able to purchase more of k at its prereform price under a quantity stabilization than they actually purchased prior to reform. This feature of a quantity stabilization raises problems, but the government's shortage of information leaves it little choice; if consumers do not retain the right to buy up to the α_i levels they might have used ex ante, they may end up worse off.

Consumer i in a quantity stabilization has three sources of income: endowment sales, firm profits earned from any ownership shares distributed by the government, and arbitrage profits or losses from purchases and sales at the prices $\overline{p}$. The firm ownership shares of consumer i, which can include a distribution of shares of firms previously owned by the state, are denoted $\theta_i = (..., \theta_{ij}, ...) \geq 0$.[7] The profile of ownership shares, $\theta = (..., \theta_i, ...)$, must satisfy $\sum_i \theta_{ij} = 1$ for each firm j. Letting p denote the postreform price vector and π the vector of firms' profits, the sum of i's endowment and profit income equals $p \cdot e_i + \theta_i \cdot \pi$. When obligations are strong overall, i's arbitrage profits equal $(p - \overline{p}) \cdot \alpha_i$, as in section 2. For an additional case of how to calculate arbitrage profits, when obligations are weak overall, see the appendix. Let $I_i(p, \theta_i)$ denote consumer i's income. So, if obligations are strong overall, $I_i(p, \theta_i) = p \cdot e_i + \theta_i \cdot \pi + (p - \overline{p}) \cdot \alpha_i$. Consumer i maximizes $u_i(x_i)$ subject

[7] Even though production sets show constant returns, we now need to specify ownership shares since firms can make arbitrage profits from their exchanges with consumers; if the government budget is balanced, these profits must be transferred back to consumers for an equilibrium to exist.

to $p \cdot x_i \leq I_i(p, \theta_i)$ and $x_i \geq 0$. We represent the solution excess demands $x_i - e_i$ by the function $z_i(p, I_i(p, \theta_i))$.

Any firm j that received a transfer τ_j from the government prior to reform must continue to receive τ_j; otherwise j might go bankrupt. Letting y_j denote firm j's production under the quantity stabilization, j's profits π_j will therefore equal the sum of its operating profits $p \cdot y_j$, its transfer τ_j, and its arbitrage profits or losses. For obligations that are strong overall, j's arbitrage profits equal $-(p - \overline{p}) \cdot \beta_j$ (see the appendix for the weak-overall case). Since j's arbitrage profits are lump sum (constant as a function of y_j), maximization of π_j reduces to maximization of $p \cdot y_j$.

Definition 2 *A quantity-stabilized equilibrium is a p, a $y_j \in Y_j$ for each firm j, and a distribution of shares θ, such that*

$$\sum_i z_i(p, I_i(p, \theta_i)) = \sum_j y_j, \text{ and} \tag{3.6}$$

$$\text{for each firm } j : p \cdot y_j \geq 0, \text{ and } y'_j \in Y_j \Rightarrow p \cdot y_j \geq p \cdot y'_j. \tag{3.7}$$

A quantity-stabilized equilibrium must be Pareto optimal; any variation in any agent's net trade is valued using the same price vector p, and so the standard proof of the first welfare theorem applies. Furthermore, as we argued in section 2, each consumer i is at least as well off at a quantity-stabilized equilibrium where obligations are strong overall compared to a prereform equilibrium in which i is exactly observed. The same conclusion holds when the commodities consumers purchase prior to reform – call these *consumption goods* – are only partially observed, as long as consumers are exactly observed prereform in the commodities they sell – call these *factors* – and obligations for factors are strong under reform. If all agents retain the right to repeat their prereform factor sales and have the option to repeat their consumption purchases then no agent can be worse off. Moreover, exact observations of factor sales are more plausible than exact observations of consumption purchases, particularly in planning applications where the state dictates factor deliveries. We state the Pareto improvement property as a proposition, but omit the straightforward proof.[8] Consumer i *sells* k prior to reform if $z_i(k) < 0$ at the prereform equilibrium.

[8] Notice that prior to reform there are no distributed profits (either due to constant returns to scale)

Proposition 2 *If each consumer i is exactly observed prior to reform in any good that i sells and those goods are subject to strong obligations under a quantity stabilization, then i cannot be worse off in a quantity-stabilized equilibrium than in the prereform equilibrium.*

Proposition 3 below reports that quantity-stabilized equilibria exist when agents are exactly observed and the government budget is balanced. Lau et al. (1997) argue that equilibria exist when in addition obligations are strong overall.[9] We can let obligations be weak without endangering existence of equilibrium or the achievement of a Pareto improvement by restricting the hunt for equilibrium prices to p such that $p \geq \overline{p}$: all agents then invoke their rights to repeat their prereform purchases. Then, even with weak obligations, quantity stabilizations exist that harm no agent.

As with any existence argument, quantity stabilizations require various technical conditions to ensure that demand and supply functions are continuous. Specifically, we now assume that each u_i is strictly quasiconcave and strictly increasing in each good and that each Y_j is a convex, closed cone that contains the negative orthant, and intersects the positive orthant only at 0.

Proposition 3 *If in the prereform economy all agents are exactly observed and the government budget is balanced, then, whether obligations for any good are weak or strong, there exist quantity-stabilized equilibria where no consumer is worse off than in the prereform equilibrium.*

The proof of Proposition 3 is in the appendix.

So, for private-ownership economies, if the government can exactly observe agents' prereform behavior and some ex ante inefficiency is present, Proposition 3 implies that the government can engineer a Pareto-improving quantity stabilization in which only weak obligations are imposed.

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or because the economy is planned. The distribution of profits post-reform can therefore only increase a consumer's welfare.

[9] The Lau et al. result (1997, Proposition 1) is not correct since it ignores the discontinuity of demand that can occur when an agent's consumption prior to reform (which is in effect the agent's endowment under a quantity stabilization) is on the boundary of his/her consumption set. Since it is unreasonable to suppose that prereform consumption is strictly positive in every coordinate, we will instead get existence by assuming that utilities are increasing. One may establish the feasibility of quantity stabilizations under conditions weaker than our increasingness assumption – see Grandmont and McFadden (1972) for a more general approach to existence that could be applied here.

A general-equilibrium quantity stabilization would presumably be implemented via a decentralized repetition of trades rather than through government-imposed lump-sum transfers – the latter simply presupposes too massive a centralization of information by a single authority. For the obligation to repeat trades to be enforceable, the government or the court system must be able to verify claims about prereform trades.

Proposition 3 assumes agents are exactly observed ex ante. If the preexisting distortion is planning, that assumption means that rationing is exact: the constraints in (3.1) hold with equality. Exact rationing is not a good description of real-world planned economies, since consumers would not have the latitude to make even trivial consumption decisions. Moreover, we will see momentarily that Pareto-improving quantity stabilizations can fail to exist if prereform rationing is only partial. But even if we suspend doubt about the plausibility of exact rationing, it is generically inconsistent with another assumption of Proposition 3, the requirement that the prereform government budget is balanced. As we saw earlier, if consumers are exactly rationed under planning and can afford their mandated consumption bundle, then, except in fluke cases, they accumulate money balances and hence the government must run a deficit. But since rationing constraints are removed under a quantity stabilization, a government deficit would lead the aggregate demand for goods to outstrip supply.[10]

The government must therefore levy enough taxes to cover its preform deficit, $\sum_j \tau_j$. In the case of planning, the government could fill the revenue gap without endangering the Pareto improvement result or relying on detailed information about individual agents by retaining the firm shares. If to preserve firm independence the government judges that it must not retain shares, then the nonexistence problem persists. The state could instead impose a lump-sum tax of up to $|\overline{p} \cdot \alpha_i|$ on each individual i. Consumers can certainly afford these levies, but the tax bills would utilize information (the α_i) that government presumably would not have immediate access to. As we argued, the government's lack of immediate access to the α_i does not by itself make quantity stabilizations impossible since indirect

[10] Since profits are distributed to consumers and since the sum of the quantity stabilization transfers equals 0, net expenditures by consumers equals the sum of firm profits plus the sum of government transfers: $\sum_i p \cdot z_i(p, I_i) = \sum_j p \cdot y_j + \sum_j \tau_j$. Consequently, if the government budget is not balanced ($\sum_j \tau_j > 0$), then $\sum_i p \cdot z_i(p, I_i)$ will not equal $\sum_j p \cdot y_j$, contradicting (3.6).

access using court verification might still be possible; but the effect of this informational gap on the deficit may be fatal.

Putting aside the deficit problem, which in any event is a difficulty only for planning applications, why is it that government ignorance of agents' prereform purchases by itself threatens the feasibility of quantity stabilizations? If the government only partially observes agents' prereform purchases, then to achieve a Pareto improvement the government must allow any consumer i to repeat purchases up to the observation level α_i at the prereform prices $\overline{p}$. Of course to avoid forcing agents to buy extra quantities of consumption goods, the government must let obligations for the goods that consumers buy be weak. To make sure there are no further obstacles to achieving a Pareto improvement, suppose that prereform consumers are exactly observed with respect to the goods they sell, and that obligations for these goods are strong under the reform. Then Proposition 2 implies that each consumer is no worse off under a quantity stabilization than with his prereform allocation, and thus a Pareto improvement would be achieved.

The difficulty with quantity stabilizations when the government lacks information about prereform purchases is instead with existence: if in a quantity stabilization the prices of consumption goods (goods k where $\alpha_i(k) > 0$) are high relative to their prereform levels, then consumers will buy these goods up to their observation constraints, thus bankrupting firms that sell those goods. But if on the other hand the quantity-stabilization prices of consumptions goods are low, then consumers will buy none of them at the prereform prices, which can bankrupt firms with obligations to buy factors at prereform prices. This bankruptcy problem, a direct outgrowth of the government's information shortage, is the main obstacle facing quantity stabilizations. The following example illustrates.

Example 2 Suppose there are two produced goods 1 and 2, each produced by a separate set of firms, and two labor inputs l_a and l_b. Before reform and under the quantity stabilization, the good 2 producers efficiently use a linear activity that employs 1 unit of l_a to produce 1 unit of good 2. Good 1 producers under the quantity stabilization use a linear activity that employs 1 unit of l_a to produce 1 unit of good 1 but ex ante they operate inefficiently (or have a different inferior technology). Specifically, in the prereform equilibrium, the inefficient good 1 producers in the aggregate purchase 2 units of l_a and 1 unit of l_b and sell

1 unit of good 1. As a matter of interpretation, think of the backward technology used ex ante by good 1 producers as employing a specialized input l_b, which becomes redundant postreform. Prereform the good 2 producers in the aggregate purchase 1 unit of l_a and sell 1 unit of good 2. Consumers are endowed only with l_a and l_b both of which they supply inelastically: prereform consumers in the aggregate sell 3 units of l_a and 1 unit of l_b and buy one unit each of goods 1 and 2. The prereform prices are $\overline{p}_1 = 3$, $\overline{p}_2 = 1$, $\overline{w}_a = 1$, $\overline{w}_b = 1$ and there are no transfers to firms. Partial observation levels of consumer purchases of the two produced goods sum to $\sum_i \alpha_i(1) \geq 1$ and $\sum_i \alpha_i(2) > 1$. All other trades are exactly observed.

We consider quantity stabilizations where the produced goods are subject to weak obligations and l_a and l_b are subject to strong obligations. Profit maximization requires that $w_a \geq p_1$ and $w_a \geq p_2$. Suppose preferences are such that consumers in any quantity-stabilized equilibrium consume a positive quantity of good 2: so then $w_a = p_2$. We must have $p_2 \leq \overline{p}_2 = 1$ since otherwise consumers will invoke their right to buy $\sum_i \alpha_i(2) > 1$ units of good 2 at price $\overline{p}_2$. Given that $\overline{w}_a = \overline{p}_2$, good 2 producers would then make negative profits even if they invoked their right to buy 1 unit of l_a for $\overline{w}_a$. We also have $w_a = 1$: if $w_a > 1$ then $w_a = p_2$ implies $p_2 > 1$ while if $w_a < \overline{w}_a$ then owners of l_a would invoke their right to sell to good 2 producers and, since good 2 producers cannot force agents to buy good 2 at price $\overline{p}_2$, good 2 producers would then make negative profits. Since $w_a = 1$, $p_1 \leq 1$. Since input l_b is now superfluous but is inelastically supplied, market-clearing requires that $w_b = 0$. Hence owners of l_b invoke their right to sell to good 1 producers at price $\overline{w}_b = 1$. Given that $p_1 \leq 1$, good 1 producers then make negative profits. ■

To sum up, for quantity stabilizations to be feasible as a reform tool for an entire economy, the government must directly or indirectly know the exact trades, agent by agent, that occur prior to reform. While such an assumption might be plausible in a planned economy, the government of a planned economy with exact rationing will generically run a deficit, and hence a quantity stabilization will require additional taxes to close the deficit. If the government indeed knows all prereform trades, then it could devise the required taxes without interfering with the achievement of a Pareto improvement, but calculation of the taxes pushes the assumption of exact observations very hard. We conclude therefore that quantity stabilizations are best suited to partial-equilibrium settings, such as the rent control example discussed in section 2.

4 Conclusion: the interpretation of Pareto improvements and production efficiency

We have argued that a policymaker can gather the information needed for a quantity stabilization simply by observing exchanges at a earlier stage in time. We have supposed implicitly that two economies, which are near or exact replicas, operate at two successive dates: the policymaker observes the date 1 economy and uses this information to set the date 2 economy's policies. The Pareto improvements we model therefore involve comparing the welfare of the date 2 agents with the welfare of the date 1 agents (as opposed to comparing the effect of a change in policies on the welfare of agents at a single date).

If the date 2 agents are simply date 1 agents at a later point in time, two questions arise. First, are quantity stabilizations somehow undermined if the date 1 agents anticipate the influence of their actions on the date 2 policy decisions? Agent i's date 1 actions certainly affect a date 2 quantity stabilization since i's date 2 transfer, $(p-\overline{p})\cdot\overline{z}_i$, is in part determined by his date 1 demand $\overline{z}_i$. (The effect of an individual agent's date 1 demands on $\overline{p}$, on the other hand, is presumably small in a large economy.) But the influence of demands on transfers does not threaten the conclusion that the date 2 allocation Pareto-improves on the date 1 allocation; it simply means that the date 1 allocation is now an endogenous variable.

Second, do the Pareto improvements we have described necessarily leave each agent i better off following a policy change if we view i's welfare as a function of his or her allocation on both dates taken together? In the case of planning at least, the answer is clear. If in the absence of reform at either date the government dictates that agent i will consume the same bundle $\overline{x}_i$ at both dates, then in the presence of reform at date 2, i will consume $\overline{x}_i$ at date 1 and a bundle at least weakly preferred to $\overline{x}_i$ at date 2. So, assuming i's overall welfare is an increasing function of date 2 utility, i will be weakly better off in this expanded sense.

Finally, consider the incentive for firms to operate efficiently under quantity stabilizations.

Firms that ex ante operated inefficiently receive a lump-sum payment under a quantity stabilization that will allow them, if they so choose, to continue to produce as they did previously; their lump-sum payments will necessarily cover their losses and keep them afloat. This would not be the profit-maximizing decision but in some applications firms may not be full-fledged profit maximizers that serve only the interests of their shareholders. Under regulation or planning, for example, firms may have long been driven by political imperatives or managed by administrators who want to preserve their jobs, and these practices can die hard. Quantity stabilization can therefore perpetuate these entrenched inefficiencies. But notice that at least the quantity stabilization reform of rent control, discussed in section 2, does not suffer from this defect since there is no production. Partial-equilibrium exchange settings therefore appear to be the most propitious environments for quantity stabilizations. In production settings, classical laissez-faire reform policies do not distribute lump-sum subsidies and hence do not shelter inefficiencies in production, but of course they will not normally deliver Pareto improvements. Price stabilizations (Mandler (1999, 2001)) do achieve Pareto improvements, by keeping relative prices at their prereform levels, but since like classical policies they do not pay out lump-sum subsidies they do not shield inefficient non-profit-maximizing firms from bankruptcy.

A Appendix

A.1 Derivation of arbitrage profits

Suppose that obligations are weak overall (the strong overall case is discussed in the text). If $p(k) > \overline{p}(k)$ and $\alpha_i(k) > 0$, consumer i will buy good k at price $\overline{p}(k)$ and resell at $p(k)$: i's arbitrage profit will then be $(p(k)-\overline{p}(k))\alpha_i(k)$. If $p(k) > \overline{p}(k)$ and $\alpha_i(k) < 0$, other agents will exercise their option to buy from i leading i to have the return $\max[p(k)-\overline{p}(k), 0]\alpha_i(k) < 0$. If $p(k) < \overline{p}(k)$ and $\alpha_i(k) < 0$, other agents will refuse to buy good k from i and if $p(k) < \overline{p}(k)$ and $\alpha_i(k) > 0$, i will refuse to buy k from other agents. So i's total arbitrage profits equal $\sum_{k=1}^{n} \max[p(k) - \overline{p}(k), 0]\alpha_i(k)$.

As for a typical firm j, if $p(k) > \overline{p}(k)$ and $\beta_j(k) > 0$, other agents will invoke their right to buy from j, and j's arbitrage return will therefore be $-(p(k)-\overline{p}(k))\beta_j(k)$. When $\beta_j(k)<0$ and $p(k)>\overline{p}(k)$, j will buy k at price $\overline{p}(k)$ and resell at $p(k)$, leading to arbitrage profits of $-(p(k)-\overline{p}(k))\beta_j(k)$. The cases where $p(k)<\overline{p}(k)$ again induce no transactions. Summing, firm j receives total arbitrage profits of $-\sum_{k=1}^{n}\max[p(k)-\overline{p}(k),0]\beta_j(k)$.

A.2 Proof of Proposition 3

We employ a standard tool, the social equilibrium existence technique (see the Debreu (1982) survey), that proceeds by setting (1) a truncated budget set for each consumer i that excludes only infeasible vectors, that is convex and compact for any $p\in\Delta_+^{n-1}=\{p\in\mathbb{R}_+^n:\sum_{k=1}^n p_k=1\}$, and that is continuous as a correspondence of p at any p such that $I_i(p,\theta_i)>0$, and (2) a truncated production set for each firm j that excludes only infeasible vectors and that is convex and compact. Using a fixed point argument, the details of which we omit, it follows that there exists a $(p,\{z_i\},\{y_j\})$ where $p\in\Delta_+^{n-1}$ such that (i) if $I_i(p,\theta_i)>0$, then z_i gives consumer i at least as much utility, given prices p, as any other point in i's truncated budget set, and (ii) the supply vector y_j gives firm j as least as much profit, given prices p, as any other point in j's truncated production set. As long as Walras' law is satisfied, we may then conclude that $\sum_i z_i=\sum_j y_j$. It is sufficient to establish the continuity of i's budget correspondences only at p such that $I_i(p,\theta_i)>0$ since we may assign i a set of pseudo excess demand vectors equal to i's entire truncated budget set whenever $I_i(p,\theta_i)=0$, thereby preserving the upper hemicontinuity of the demand correspondence. As we will see, our assumptions imply that the $(p,\{z_i\},\{y_j\})$ we find must satisfy $p>>0$. Since $e_i+\alpha_i>0$ for each i, $p>>0$ implies that no i has $I_i(p,\theta_i)=0$ at $(p,\{z_i\},\{y_j\})$ and hence the pseudo excess demands are irrelevant. Also, since only infeasible points are truncated from the choice sets, one may show that the excess demands z_i and supplies y_j remain optimal when agents are free to choose from their original, nontruncated choice sets (we omit the details of this step too).

Consider strong overall obligations first. To meet condition (2) above, let $\widetilde{Y}_j$ denote the intersection of Y_j and a sufficiently large closed rectangle in $\mathbb{R}^n$. For each $p \in \Delta_+^{n-1}$, define

$$\mathit{\Pi}_j(p) = \max_{y_j \in \widetilde{Y}_j} p \cdot y_j - (p - \overline{p}) \cdot \beta_j + \tau_j.$$

Since $\beta_j = \gamma_j$ when agents exactly observed prior to reform, $\mathit{\Pi}_j(p)$ is j's maximum level of profits at prices p assuming obligations are strong overall (and that j has $\widetilde{Y}_j$ as its production set). Setting $y_j = \gamma_j$,

$$p \cdot \gamma_j - (p - \overline{p}) \cdot \beta_j + \tau_j = \overline{p} \cdot \beta_j + \tau_j \geq 0,$$

where the inequality follows from (3.5). Hence $\mathit{\Pi}(p) = (..., \mathit{\Pi}_j(p), ...) \geq 0$.

Fix an arbitrary distribution of shares θ. When obligations are strong overall, consumer i's budget constraint at prices p is $p \cdot z_i \leq (p - \overline{p}) \cdot \alpha_i + \theta_i \cdot \mathit{\Pi}(p)$, or equivalently,

$$p \cdot x_i \leq p \cdot (e_i + \alpha_i) - \overline{p} \cdot \alpha_i + \theta_i \cdot \mathit{\Pi}(p) = I_i(p, \theta_i). \tag{A.1}$$

Since $\overline{p} \cdot \alpha_i \leq 0$, $I_i(p, \theta_i) \geq 0$. To meet condition (1), intersect i's budget set $\{x_i \in \mathbb{R}_+^n : p \cdot x_i \leq I_i(p, \theta_i)\}$ with a large closed rectangle in $\mathbb{R}^n$, thus generating a truncated budget set that is compact and a continuous correspondence of p whenever $I_i(p, \theta_i) > 0$.

The fixed point argument then establishes that there is a $(p > 0, \{z_i\}, \{y_j\})$ such that, for each firm j, y_j is optimal for j at p, and, for each consumer i with $I_i(p, \theta_i) > 0$, z_i is optimal for i at p. We show that $I_i(p, \theta_i) > 0$ for all i; so then z_i is optimal at p for each i. Given that (a) $p > 0$, (b) $\sum_i (e_i + \alpha_i) >> 0$, (c) $\overline{p} \cdot \alpha_i \leq 0$ for all i, and (d) $\mathit{\Pi}(p) \geq 0$, at least one agent k must have $I_k(p, \theta_k) > 0$ – see (A.1). Since u_k is increasing in each good, it must be that $p >> 0$; otherwise z_k would not be optimal at p. Our assumption that $e_i + \alpha_i > 0$ for each i then implies that each $I_i(p, \theta_i) > 0$.

Finally, to show that markets clear, we confirm that Walras' law holds at $(p, \{z_i\}, \{y_j\})$, i.e., $\sum_i p \cdot z_i - \sum_j p \cdot y_j = 0$. Using the agent budget constraints and the definition of firm profits,

$$\sum_i p \cdot z_i - \sum_j p \cdot y_j = \sum_i [(p - \overline{p}) \cdot \alpha_i + \theta_i \cdot \pi] - \sum_j [\pi_j + (p - \overline{p}) \cdot \beta_j - \tau_j].$$

Since $\sum_i \alpha_i = \sum_j \beta_j$ and the government budget is balanced, $\sum_i p \cdot z_i - \sum_j p \cdot y_j = 0$, as desired. We conclude that $(p, \{y_j\}, \theta)$ is an equilibrium.

Next consider obligations that are weak for an arbitrary subset of goods. Again fix the distribution of shares θ. Define the function $\overline{\lambda} : \mathbb{R}^n_+ \to \mathbb{R}$ by $\overline{\lambda}(p) = \arg\max_\lambda \lambda\overline{p}$ s.t. $\lambda\overline{p} \leq p$ and $\lambda \leq 1$. If prereform prices were to equal $\overline{\lambda}(p)\overline{p}$ and reform prices were to equal p, then agents would invoke all of their prereform rights to buy goods and i's arbitrage profits would equal $(p - \overline{\lambda}(p)\overline{p}) \cdot \alpha_i$. Defining $\widetilde{Y}_j$ as before, let $\widetilde{\mathit{\Pi}}_j(p) = \max_{y_j \in \widetilde{Y}_j} p \cdot y_j - (p - \overline{\lambda}(p)\overline{p}) \cdot \beta_j + \overline{\lambda}(p)\tau_j$. Given that the function $\overline{\lambda}$ is continuous, i's truncated budget correspondence remains a continuous correspondence of p whenever the right hand side of the budget inequality

$$p \cdot x_i \leq p \cdot (e_i + \alpha_i) - \overline{\lambda}(p)\overline{p} \cdot \alpha_i + \theta_i \cdot \widetilde{\mathit{\Pi}}_j(p)$$

is strictly positive. Hence, just as in the strong overall obligations case, there exists a $(p^* >> 0, \{y_j^*\}, \theta)$ such that if prereform prices equaled $\overline{\lambda}(p^*)\overline{p}$ and transfers to firms equaled $\overline{\lambda}(p^*)\tau_j$, then $(p^*, \{y_j^*\}, \theta)$ would be a quantity-stabilized equilibrium. Since $p^* >> 0$, $\overline{\lambda}(p^*) > 0$. We therefore have $\mathit{\Pi}_j(\frac{1}{\overline{\lambda}(p^*)}p^*) = (\frac{1}{\overline{\lambda}(p^*)})\widetilde{\mathit{\Pi}}_j(p^*)$. Each i's budget set at postreform prices $\frac{1}{\overline{\lambda}(p^*)}p^*$, prereform prices $\overline{p}$, and firm subsidies τ_j is therefore identical to the budget set that occurs with reform prices p^*, prereform prices $\overline{\lambda}(p)\overline{p}$, and firm subsidies $\overline{\lambda}(p)\tau_j$. Since in addition each y_j^* is profit-maximizing at prices $\frac{1}{\overline{\lambda}(p^*)}p^*$, $[\frac{1}{\overline{\lambda}(p^*)}p^*, \{y_j^*\}, \theta]$ is a quantity-stabilized equilibrium.

The arguments given in section 2 imply that the equilibria are Pareto improving.

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